This is the first of a two-part article covering the use of simple springs to model soil behavior. Covered in this article are suggestions for spring placement and procedures for calculating spring stiffness and ultimate spring strength. Covered in the second part of this article (to be published in the June 2014 issue) are the use of springs in a plane-frame structural analysis and determination of the ultimate lateral capacity of a post or pier foundation. Part 2 also includes an overview of safety factors for allowable stress design and resistance factors for load and resistance factor design.

The interaction between an embedded post or pier foundation and the surrounding soil is a complex, three-dimensional problem that is simplified for structural analysis. When assessing this interaction, designers are interested in two different but related phenomena. First is the deformation of the soil as load is applied to the soil by the foundation system. Second is the ability of the soil to resist the applied load without failing. These two phenomena are herein referred to as soil stiffness and soil strength.

In assessing the effects of lateral forces applied to the soil by a shallow post or pier foundation, the latest version of ANSI/ASAE EP486.2 takes two different approaches. The first approach uses the set of equations published in the document. This approach is referred to as the simplified method because it does not require any special computer software, just a basic calculator. In many respects, this approach could be referred to as the traditional method because it mirrors past procedures.

The second approach relies on modeling soil with a series of simple springs. This approach requires structural analysis software and is referred to as the universal method. Modeling soil behavior with simple springs is a discrete approach to analysis that has been used for well over a century. A summary of foundation-soil interaction models developed by researchers who have used this discrete approach has been provided by Maheshwari (2011). Within the post-frame building community, McGuire (1998) used a spring model to study the behavior of nonconstrained posts subjected to ground-line shear forces and ground-line bending moments applied such that they caused below-grade post rotation in opposite directions (see Load Case B in Figure 1). McGuire conducted his investigation to illustrate that when shear and bending moments are so applied, most equations used to calculate allowable embedment depth are not applicable.

**When to Model with Soil Springs**

The modifier universal used in ANSI/ASAE EP 486 was given to the soil spring method because the method can be used without restriction. Conversely, use of the simplified method assumes the following:

1. At-grade pier/post forces are not dependent on below-grade deformations.
2. The below-grade portion of the foundation has an infinite flexural rigidity ($EI$).
3. Soil is homogeneous for the entire embedment depth.
4. Soil stiffness either is constant for all depths below grade or linearly increases with depth below grade.
5. Width of the below-grade portion of the foundation is constant. This generally means that there are no attached collars or footings that are effective in resisting lateral soil forces.

The second of the preceding simplify-
ing assumptions—that the below-grade portion of the foundation is infinitely stiff—is assumed to hold where soil stiffness is assumed to increase linearly with depth and:

$$d < 2\left(\frac{EI}{(2AE_s)}\right)^{0.20}$$

or where soil stiffness is assumed constant with depth and:

$$d < 2\left(\frac{EI}{(2E_s)}\right)^{0.25}$$

where $d$ is depth of embedment; $EI$ is flexural rigidity of the post/pier foundation; $E_s$ is Young’s modulus of the soil; and $A_s$ is the linear increase in Young’s modulus with depth below grade.

When a post/pier foundation does not comply with one of the five conditions associated with application of the simplified method, consideration should be given to using the universal method with its soil springs. In some cases, the results will be significantly different.

**Placement of Soil Springs**

Figure 2a illustrates the use of soil springs to model a nonconstrained post in a multilayered soil. Figure 2b shows the modeling of a nonconstrained post that has an attached footing and an attached collar. In this case individual springs are required for both the footing and the collar because each has different widths relative to the post.

Figure 3 shows an embedded post that abuts a slab-on-grade (i.e., a surface-constrained post). To model the restraint that the slab provides when the post moves toward the slab, the slab is modeled as a vertical roller support (Figure 3a). Because the slab abuts only the inside of the post and is not attached to the post, it does not apply a force to the post when the post moves away from the slab; thus it is modeled as a nonconstrained post (Figure 3b).

A closer spring spacing enables more accurate estimation of post/pier forces and soil pressures. ANSI/ASAE EP486.2 recommends that soil-spring spacing, $t$, not exceed $2w$ where $w$ is the side width of a rectangular post/pier and diameter of a round post/pier. (Note: the side of a rectangular post or pier is the surface perpendicular to the loaded face.) Generally, at least five springs should be used.

**Soil Spring Stiffness**

All springs are assumed to exhibit linear-elastic behavior until a point of soil failure is reached, at which point the force in the soil spring stays constant as the spring undergoes additional deformation. A graphical depiction of this behavior is shown in Figure 4.

The initial stiffness, $K_{ih}$, of an individual soil spring located at depth $z$ is given as

$$K_{ih} = 2.0 t E_s$$

(1)

where

- $t$ = thickness of the soil layer represented by the spring, in.
- $E_s$ = Young’s modulus for soil at depth $z$, lbf/in$^2$

ANSI/ASAE EP486.2 contains equations for calculating $E_s$ from laboratory test results, prebored pressuremeter test (PMT) results, cone penetration test (CPT) results, standard penetration test (SPT) results, and undrained soil shear
strength. In the absence of such test data, ANSI/ASAE EP486.2 allows use of the presumptive values in Table 1 for silts and clays and those in Table 2 for sands and gravels. It is important to note that $E_s$ is assumed to be constant with depth for silts and clays and to increase linearly with depth for sands and gravels. To calculate $E_s$ at a particular depth in sands and gravels, multiply the $A_E$ value in the far right column of Table 2 by depth, $z$. In equation form:

$$E_{s,z} = E_s z$$  \hspace{1cm} (2)

where

$$\begin{align*}
E_{s,z} &= E_s \text{ that is equal to zero at grade and increases linearly with depth } z \text{ below grade} \\
A_E &= \text{ increase in Young's modulus per unit increase in depth } z \text{ below grade, lbf/in}^2 \text{ (kN/m}^2) \\
z &= \text{ depth below grade, in (m)}
\end{align*}$$

For a post that is driven into the ground or a helical pier that is turned into the ground, the material surrounding the post/pier at a given depth will have fairly uniform properties within several feet of the post/pier. This often is not the case for a post/pier that is placed in an augered hole that is backfilled with a different soil. When soil backfill has properties different from those of the surrounding soil, Young's modulus $E_s$ for soil at depth $z$ can be calculated as

$$E_s = \frac{1}{I_s} \left( \frac{E_{s,B}}{E_{s,U}} + (1-I_s) \right)$$  \hspace{1cm} (3a)

$$E_s = E_{s,B}$$  \hspace{1cm} (3b)

$$E_s = E_{s,U}$$  \hspace{1cm} (3c)

where

$$\begin{align*}
I_s &= \text{ strain influence factor, dimensionless} \\
J &= \text{ distance (measured in the direction of lateral foundation movement) between the edge of the backfill and the face of the foundation component at depth } z \text{ (see Figure 5)} \\
b &= \text{ width of the post/pier, collar, or footing that is surrounded by the backfill at depth } z
\end{align*}$$

The strain influence factor is the fraction of total lateral displacement that is due to soil straining within a distance $J$ of the face of the foundation.

The condition of $J = 0$ (and thus $E_s = E_{s,U}$) would apply to a driven pier/post for which the foundation is entirely surrounded by unexcavated soil. When a pier/post is entirely backfilled with concrete or controlled low-strength material (CLSM), $E_s$ is simply equated to the Young's modulus for the soil surrounding the concrete or CLSM.

### Table 1. Presumptive Properties for Silt and Clay (Cohesive) Soils

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Consistency</th>
<th>Moist unit weight, $\gamma$</th>
<th>Undrained soil shear strength, $S_u$</th>
<th>Young's modulus for soil, $E_s$</th>
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<td>Medium to Stiff</td>
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<tr>
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<td>Very Stiff to Hard</td>
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<td>14</td>
<td>8400</td>
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<td>Medium to Stiff</td>
<td>120</td>
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<td>8400</td>
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<td></td>
<td>Very Stiff to Hard</td>
<td>120</td>
<td>14</td>
<td>4480</td>
</tr>
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</table>

(a) Loading assumed slow enough that sandy soils behave in a drained manner.
(b) Estimate of stiffness at rotation of 1° for use in approximating structural load distribution. For evaluation of serviceability limit state, use values that are 1/3 of tabulated value.
(c) Constant values of stiffness used for calculation of clay response. Stiffness increasing with depth from a value of zero used for calculation of sand response.

### Table 2. Presumptive Properties for Sand and Gravel (Cohesionless) Soils

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Consistency</th>
<th>Moist unit weight, $\gamma$</th>
<th>Drained soil friction angle, $\phi^{(c)}$</th>
<th>Increase in Young's modulus per unit depth below grade, $E_{s,B}/E_{s,U}$</th>
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</thead>
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<tr>
<td>SM, SC, SP-SM, SW-SM, SW-SC</td>
<td>Loose</td>
<td>105</td>
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<td>Medium to Dense</td>
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<td>Very Dense</td>
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<tr>
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<tr>
<td></td>
<td>Medium to Dense</td>
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<td>1320</td>
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<td></td>
<td>Very Dense</td>
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<td>1760</td>
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<td>Very Dense</td>
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<td>GW-GC, GC, SC</td>
<td>Loose</td>
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<td>35</td>
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<td>Medium to Dense</td>
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<tr>
<td></td>
<td>Very Dense</td>
<td>130</td>
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<td>2200</td>
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</table>

(a) Rapid undrained loading will typically be the critical design scenario in these soils. Laboratory testing is recommended to assess clay friction angle for drained loading analysis.
(b) Estimate of stiffness at rotation of 1° for use in approximating structural load distribution.
(c) Constant values of stiffness used for calculation of clay response. Stiffness increasing with depth from a value of zero used for calculation of sand response.
(d) Assumes soil is located below the water table. Double the tabulated $A_E$ value for soils located above the water table.
Soil Spring Strength

The ultimate load that an individual spring can sustain is given as

\[ F_{ult} = p_{uz} \times b \times t \]  

(4)

where

- \( F_{ult} \) = ultimate load that an individual spring at depth \( z \) can sustain, lbf
- \( p_{uz} \) = ultimate lateral soil resistance for unexcavated soil at depth \( z \), lbf/ft^2
- \( b \) = width of the face of the post/pier, footing or collar that applies load to the soil when the foundation moves laterally, ft
- \( t \) = thickness of a soil layer that is represented with a soil spring with stiffness \( K_{SP} \), ft
- \( z \) = distance of spring below grade, ft
- \( y_z \) = moist unit weight of soil, lbf/ft³

Alternatively, \( p_{uz} \) can be calculated for cohesionless soils (sands and gravels) as

\[ p_{uz} = 3 \sigma_{vz} K_p = 3 (y_z - u_z) K_p \]  

(5)

and for cohesive soils (silt and clays) as

\[ p_{uz} = 3 S_U (1 + z/2b) \]  

for \( 0 < z < 4b \)  

\[ p_{uz} = 9 S_U \]  

for \( z > 4b \)  

(6)  

(7)

where

- \( K_p \) = coefficient of passive earth pressure, dimensionless
- \( = (1 + \sin \phi)/(1 - \sin \phi) \)
- \( \sigma \) = soil friction angle, degrees
- \( \sigma_{vz} \) = effective vertical stress at depth \( z \), lbf/ft²
- \( u_z \) = pore water pressure at depth \( z \), lbf/ft²
- \( y_z \) = moist unit weight of soil, lbf/ft³
- \( b \) = nominal width of post/pier/footing/collar and the edge of the backfill.

Properties

When doing foundation design involving soil springs, one of the first steps is to construct a table of spring properties. For each spring this should include depth \( z \), soil layer thickness \( t \), foundation width \( b \), Young’s modulus \( E_S \) (or the increase in Young’s modulus with depth \( A_E \)), spring stiffness \( K_{SP} \), ultimate lateral soil strength \( S_U \) for clay soils, effective stress \( \sigma_{vz} \), and soil friction angle \( \phi \) for sands and gravels, ultimate lateral soil resistance \( p_{uz} \), and ultimate spring strength \( F_{ult} \).

Example 1

Foundation Description

A nominal 6- by 6-inch post is embedded 4 feet. It rests on a concrete footing. Two nominal 2- by 6-inch wood blocks, 12 inches in length, are bolted to each side of the base of the post to increase the uplift resistance and lateral strength capacity of the foundation. The top 2.5 feet of soil are classified as medium to stiff ML silts. The next several feet of soil below this clay layer are classified as medium to dense SW sands. The water table is located 7 to 8 feet below grade. For this first example, backfill is assumed to identically match the surrounding soil. A depiction of this foundation is shown in Figure 6.

Spring Placement

Three depths are associated with an abrupt change in soil and/or post design properties that will affect spring placement. The obvious two are the change in soil type at a depth of 30 inches and the change in foundation width from 5.5 inches to 12 inches at a depth of 42.5 inches. The less obvious change is that associated with the ultimate strength of clay soil. In accordance with equations 6 and 7, the ultimate strength \( F_{ult} \) of cohesive soils switches from increasing linearly with depth to remaining constant with depth at a distance \( 4b \) which is equal to 22 inches because post width \( b \) is 5.5 inches. The selected placement is shown in Figure 6.

Presumptive Properties

From Table 1, the medium to stiff ML silt has a moist unit weight \( y \) of 120 lbf/ft³, an undrained soil shear strength \( S_U \) of 7 lbf/in² and a Young’s modulus of 6160 lbf/in². From Table 2, the medium to dense SW sand has a moist unit weight \( y \) of 120 lbf/ft³, a drained soil friction angle \( \phi' \) of 35 degrees and an increase in the Young’s modulus with depth \( A_E \) of 220 (lbf/in²)/in. With respect to the latter, the table value of 110 (lbf/in²)/inch is doubled in accordance with footnote (d) because the soil represented by the springs is all located above the water table.

Tabulated Values

Soil-spring stiffness and ultimate strength values are compiled in Table 3. To keep everything in consistent units, the moist unit weight of 120 lbf/ft³ is listed in Table 3 as 0.06944 lbf/in³. Depth \( z \) for each spring was automatically calculated in the spreadsheet as \( z_i = z_{i-1} + (t_i + t_{i-1})/2 \), where \( i \) is the spring number. Because the water table was located...
below all the springs, the effective vertical stress for all soil springs was numerically equal to the total vertical stress.

**Comments**

The stiffness of an individual spring is not a function of the width of the foundation element that the spring is acting upon. Consequently, a spring that will be pushing on an 18-inch-wide footing is assigned the same stiffness as one at the same depth in the same soil that is pushing on a 6-inch-wide post. Conversely, ultimate spring strength is a function of the width of the foundation element upon which the spring acts. The significant impact that this dependence can have on ultimate spring strength is evident when comparing $F_{ult}$ for springs 7 and 8 in Table 3.

**Example 2**

**Foundation Description**

This is the same foundation described in example 1 with the exception that the backfill is a mixture of the ML silt and SW sand removed by the 18-inch-diameter auger used to form the post hole. The mixture is compacted by hand in 6-inch lifts.

**Spring Placement**

The abrupt change in soil and/or post design properties that affect spring placement are the same as for example 1; thus the same spring placement is used.

**Presumptive Properties**

Properties for the unexcavated soil remain as compiled in Table 3. The mixture of approximately 2.5 feet of ML silt and SW sand removed by the 18-inch-diameter auger used to form the post hole. The mixture is compacted by hand in 6-inch lifts.
ously noted, this is because backfill does not factor into calculations of $F_{ult}$.

### Summary

The latest version of ANSI/ASAE EP486 incorporates the ability to use soil springs to model the behavior and predict the ultimate strength of shallow post/pier foundations for conditions not previously possible.

This includes situations where soil properties vary with depth and the thickness of the foundation is not constant.

This article summarized and demonstrated methods for calculating the stiffness and strength of these soil springs. In Part 2 of this article, methods for incorporating the use of springs in plane-frame structural analyses will be presented, along with special techniques used to determine the ultimate lateral capacity of a post/pier foundation.

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### References


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### Table 4. Soil Spring Properties for Example 2

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<tr>
<th>Spring number</th>
<th>Front width of foundation at spring location, b</th>
<th>Thickness of soil layer represented, c</th>
<th>Distance from surface, t</th>
<th>Increase in Young's modulus with depth, $E_{as}$</th>
<th>Young's modulus for un-excavated soil, $E_{un}$</th>
<th>Young's modulus for backfill, $E_{bf}$</th>
<th>Backfill thickness, $t_f$</th>
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